Modeling of the youBot in a serial link structure using twists and wrenches in a bond graph

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Abstract. We present a walk-through tutorial on the modeling of a complex robotic system, like the newly developed desktop mobile manipulator youBot developed by KUKA[5, 4]. The tutorial shows the design of models for typical robotic elements, done in a reusable object-oriented style. We employ an energy-based approach for modeling and its bond-graph notation to ensure encapsulation of functionality, extendability and reusability of each element of the model. The kinematic representation of mechanical elements is captured using screw theory. The modeling process is explained in two steps: first submodels of separate components are elaborated and next the model is constructed from these components.

1 Introduction

The modeling of the dynamics of a complicated robotic systems is a time consuming process. One of the ways to increase speed of development is reuse. This concept is rarely applied in modeling of robotic systems, because the mechanical parts of a robotic systems often require development of the dynamic models from scratch. Moreover, different applications require different types of modeling assumptions, even for the same robot. The concept of power-based modeling creates the possibility of representing a dynamic model in a simple way, while preserving the information about modeling decisions. This allows simple extension of the model. A bond graph is a graphical representation method of power-based modeling. Bond-graph models can be reused elegantly, because bond graphs are non-causal. The submodels can be seen as objects with defined power interfaces; bond-graph modeling is a form of object-oriented physical system modeling.

Bond-graph theory and notation are well developed and described in [3, 2]. In short, the representation is done by means of a directed graph: the vertices are submodels that describe interesting physical behavior and the edges represent energy relations. In addition to these edges, signals are used to pass information between vertices and do not represent energy exchange in the system. Bond graphs are a domain-independent representation of the model, therefore allowing easy capture the behavior of robotic systems, which often combines electrical,

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mechanical and hydraulic domains. In this paper, we will only concentrate on the mechanical behavior and only indicate how the model can be extended to other domains.

In order to develop a well structured model, the system behavior should be decomposed into concepts, idealized descriptions of physical phenomena. The concepts are combined into recognizable dominant behaviors of the tangible system parts(components). The components are used to compose the model of the robotic system.

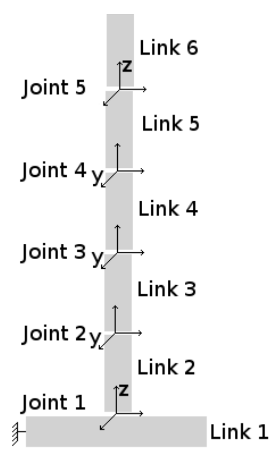
The dynamic behavior of the youBot (mobile manipulator robot) can be decomposed, without losing generalization, into three types of components: Mecanum wheels, joints of the manipulator and links of the manipulator.

Section 2 gives a description of the youBot. Sections 3 and 4 provide descriptions of the modeling assumptions and the modeling process for the links and joints of the manipulator using screw theory. Section 5 elaborates modeling of the Mecanum wheels. The last section presents the model composition process.

2 Description of the platform

For this walk-trough tutorial, the modeling of the robotic platform youBot(Fig. 1a) is used as an example. The kinematic structure of the youBot robotic manipulator is shown in Fig. 1b.

(a) youBot



(b) Schematic representation of the youBot

Fig. 1: The youBot

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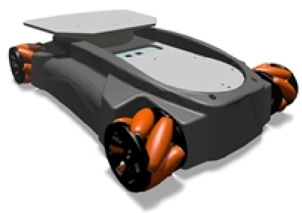
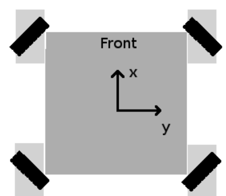
The robotic manipulator has six links connected by 5 actuated rotational joints. The axis of rotation of joint 1 and 5 is the z-axis in the frame depicted, for joint 2,3 and 4 the axis of rotation is the y-axis.

Four Mecanum wheels are mounted to the first link of the robotic actuator to make it mobile, this is shown in Fig. 2.



(a) youBot Base



(b) Schematic representation of the youBot base

Fig. 2: The youBot base with Mecanum wheels

3 Submodel of a link

A modeling process begins with defining a dominant behavior of the component. For robotic applications, the dominant behavior of the links in the manipulator is the behavior of a body to which external wrenches are applied. As long as the youBot is used within the specifications, it is valid to assume that the links are rigid and are treated as rigid bodies. However, the submodel can be adapted to include other behavior without effecting the interface with other parts of the youBot model.

3.1 Modeling rigid body dynamics of a link

The model of rigid-body dynamics consists of inertia and gyroscopic effects. According to screw theory the kinematics of a rigid-body motion can be represented as a rotation (ω) about an axis along with translation (v) along the same axis [1]. Furthermore, we will use the following notation:

- $T = \begin{pmatrix} \omega \\ v \end{pmatrix}$: twist

- $W = \begin{pmatrix} \tau \\ F \end{pmatrix}$: wrench
- $I = \begin{pmatrix} J & 0 \\ 0 & M \end{pmatrix}$: inertia tensor.
- F: force
- P: momentum screw
- τ: torque
- M: mass matrix
- J: inertia matrix

The wrench balance representing the inertia effects, expressed in the principal inertia frame k, (in the center of gravity and oriented along the three primary inertia axes) will have a form [8, 7]:

$$I^k \dot{T}_a^{k,0} = \begin{pmatrix} \tilde{P_\omega}^k & \tilde{P_v}^k \\ \tilde{P_v}^k & 0 \end{pmatrix} T_a^{k,0} + (W^k)^T \tag{1}$$

where

- k is the principal inertia frame of the body
- 0 is the inertial frame
- a is the body

In the above relation the component for an inertia can be recognized on the left hand side of the equal sign. The first component at the right hand side of the equal sign represents the fictitious forces and torques (wrenches) including the gyroscopic effect, the second component at the right hand side of the equal sign represents externally applied wrenches. A wrench balance as described by the formula above is represented in a bond graph by a 1-junction. The bond graph shown in Fig. 3 represents the equation above.

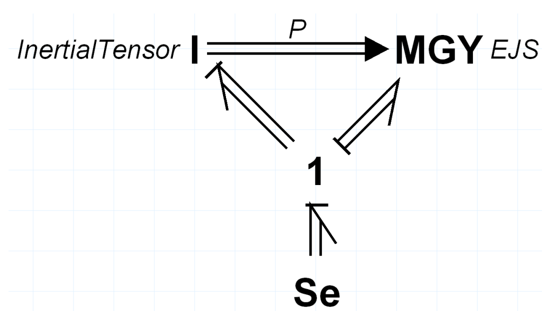


Fig. 3: A bond-graph model of a rigid body including gyroscopic effects

In this model the fictitious wrenches including the gyroscopic effects are expressed by a gyrator (MGY element). A gyrator represents a power continuous

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relation between efforts and flows. The externally applied wrenches are represented by the Se (Source of effort) element.

3.2 Externally applied wrenches

The externally applied wrenches include joint reactions, collisions, gravity etc. The most significant ones, that have a constant effect on the link dynamics, are the joints reactions and gravity. The joints will be connected to a link by means of power ports, through which the joints apply wrenches to the links. However, these wrenches are expressed in the frame that corresponds with the actuated joint. The wrench balance, as discussed in the previous section, is represented in a principal inertia frame of the body. Therefore, a transformation of coordinates is required between these parts of the model.

A homogeneous matrix H_i^j represents the position of a frame i with respect to a frame j with a translation component p and a rotation component R:

$$H_i^j = \begin{pmatrix} R_i^j & p_i^j \\ 0 & 1 \end{pmatrix} \tag{2}$$

A twist, in matrix form[7], is defined as the product of a position and the derivative with respect to time of the same position[7]:

$$\begin{aligned} \tilde{T}_i^{i,j} &= H_j^i \dot{H}_i^j \\ \tilde{T}_i^{j,j} &= \dot{H}_i^j H_j^i \end{aligned} \tag{3}$$

These two expressions can be combined into:

$$\tilde{T}_k^{j,l} = H_i^j \tilde{T}_k^{i,l} H_j^i \tag{4}$$

Which represents a change of coordinates for twists in matrix form. It is possible to see [6, 7] that a change of coordinates for twists in vector form, as applied in this paper, can be expressed as:

$$T_k^{j,l} = Ad_{H_i^j} T_k^{i,l} \tag{5}$$

with

$$Ad_{H_i^j} = \begin{pmatrix} R_i^j & 0 \\ p_i^j R_i^j & R_i^j \end{pmatrix} \tag{6}$$

the adjoint of the homogeneous matrix H_i^j.

Since a change of coordinates is power continuous, an expression for a change of coordinates for wrenches can be found:

$$W^j T_k^{j,l} = W^j Ad_{H_i^j} T_k^{i,l} = (Ad_{H_i^j}^T {W^j}^T)^T T_k^{i,l} = W^i T_k^{i,l} \tag{7}$$

such that

$$(W^i)^T = Ad_{H_i^j}^T (W^j)^T \tag{8}$$

Since the relations are power continuous, a TF element is used to represent this change of coordinates in the bond-graph language.

Coordinate transformations are required between three frames:

1. The body-fixed frame in the previous joint, from now on called frame i
2. The body-fixed frame in the next joint, from now on called frame j
3. The principal inertia frame of the link, from now on called frame k.

To relate these three frames only two changes of coordinates are required since the third change of coordinates can be composed of the other two. For example:

$$H_j^k = H_i^k H_j^i \tag{9}$$

In equation(9) the two changes of coordinates changes are applied between:

1. Frame i and frame k
2. Frame j and frame i

And frame i has a central role. The choice can be made to give either frame i, frame k or frame j this central role. This does not effect the model behavior or reusability of the model. In this paper, frame i has the central role. Therefore, the changes of coordinates are applied as shown in the example.

Since deformation of the link is neglected, it is assumed that the above mentioned changes of coordinates are constant in time. However, the model can easily be extended to include those effects by replacing the linear relation with non-linear, time-dependent ones. Fig. 4 shows the model of the link including the discussed changes of coordinates. *p* and *p1* are the power ports to which the joints will be connected.

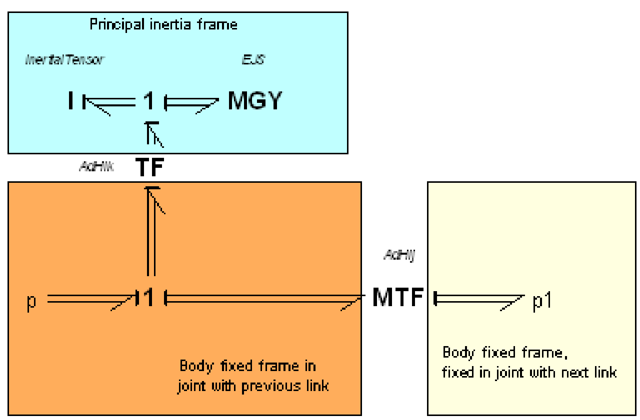

Fig. 4: A bond-graph model of a rigid body with changes of coordinates to the joints

Gravity, expressed in the inertial frame 0, is a constant force (wrench) that applies to the body in the negative z-direction and can be modelled as a Se element in the bond-graph language. The other parts of the model are not expressed

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in the inertial frame and therefore a transformation of coordinates is required between the inertial frame and one of the other frames. Since the frame in the previous joint is used as a starting point, it is chosen to apply the coordinate transformation between system 0 and frame i. The H-matrix that corresponds to this change of coordinates is passed on by the previous joint as a signal that modulates the transformations.

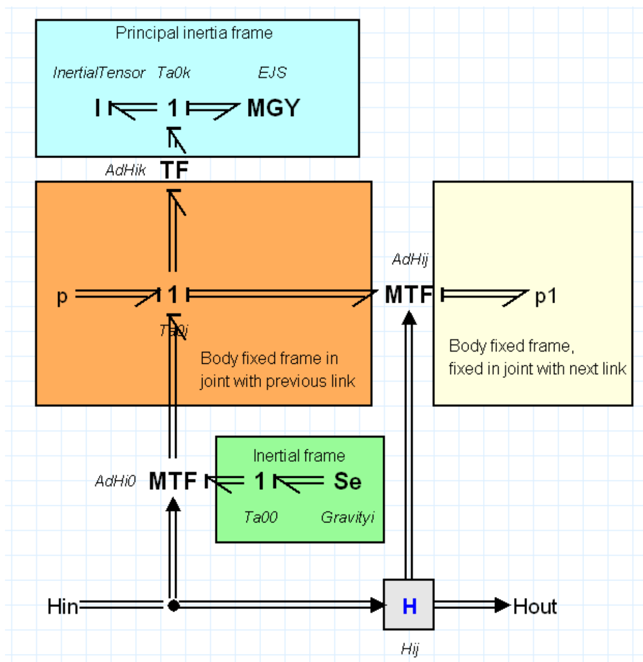

Fig. 5: A bond-graph model of a link

Fig. 5 shows the complete submodel of a link. In this model, the Se element representing gravity can be found with the coordinate transformation between the inertial frame 0 and frame i. The signal port Hin is used to obtain the position of frame i with respect to the inertial frame. This is then multiplied by H_i^j, the relative position of frame j with respect to frame i, to obtain the position of frame j with respect to the inertial frame. This position is passed on the the next joint through signal port Hout. The ports p and $p1$ are the power ports of the link, to which the joints will be connected.

4 Submodel of a joint

Just as with the submodel of a link, it has to be decided which behavior will be modelled and which behavior will be neglected. For this model, it is decided only to model the transfer of energy in a joint. Therefore, the following behavior has explicitly been neglected:

- Friction in the joint
- Energy storage in the joint

However, the model can easily be extended to include these and other effects.

Since a joint establishes an energetic connection between two links, it imposes a relation between the wrenches and the twists of the two bodies.

A wrench applied in a joint is applied between the two connected links and, in fact, applies the same wrench to both links. This specifies the relation between the wrench applied by the actuator and the wrenches applied to the connected links:

$$W_{act} = W_{link1} = W_{link2} \tag{10}$$

On the other hand, there is a relation between the twists of link 1 with respect to the inertial frame, expressed in frame n ($T_1^{n,0}$) and the twist of link 2 with respect to the inertial frame, expressed in frame n ($T_2^{n,0}$). Just as with other flow type variables the following relation holds:

$$T_2^{n,0} = T_1^{n,0} + T_2^{n,1} \tag{11}$$

The above two relations are represented by a 0-junction in the bond-graph language. For these relations to apply, all twists and wrenches should be expressed in the same frame. For this paper frame i is selected, this does not effect reusability of the model. A joint is an energetic connections between two links. The twists and wrenches of these two links are expressed in frames fixed with the link. As a result they are expressed in different frames and a change of coordinates is required.

In the relations of the joint the actuation is a wrench, while the actuator applies a torque in the joint. To transform between the actuator torque and the wrench applied between the two bodies and on the other hand, a unit wrench can be applied:

$$W = \hat{W}\tau \tag{12}$$

Where, in case of a rotational joint:

$$\hat{W} = \begin{pmatrix} \hat{\tau} \\ 0 \end{pmatrix} \tag{13}$$

and $\hat{\tau}$ is the axis around which the torque is applied.

Due to power continuity this imposes a relation between the twist in the joint and the joint velocity:

$$W^T T_a^{i,b} = (\hat{W}\tau)^T T_a^{i,b} = \tau \hat{W}^T T_a^{i,b} = \tau \dot{q} \tag{14}$$

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such that

$$\dot{q} = \hat{W}^T T_a^{i,b} \tag{15}$$

Since this set of relations is power continuous, they can be represented by a transformer in the bond-graph language. Fig. 6 shows the model of a joint.

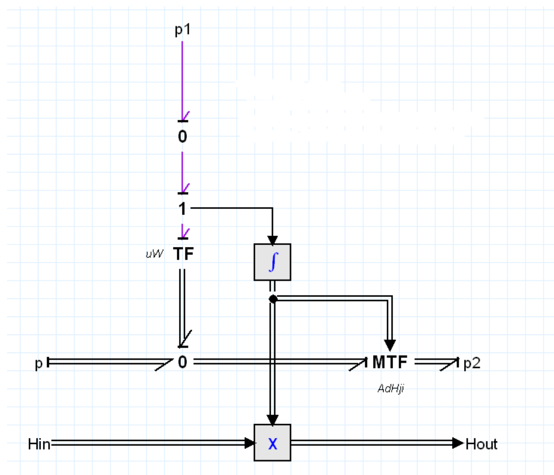

Fig. 6: A bond-graph model of a joint

The change of coordinates is represented by the MTF element "*Adji*". The conversion from joint torque to wrench and from twist to joint velocity is represented by the TF element "*uTbai*".

In the bond-graph model two other blocks are present, an $\int$-block and an x-block. The $\int$-block calculates the joint position based on an integration of the joint velocity and uses this to construct the H matrix that represents the position of the joint: H_j^i. This matrix is then used for the coordinate transformation. A multiplication block indicated by the symbol x multiplies two matrices. The two H-matrices that are multiplied are:

- The position of frame i with respect to the inertial frame 0, obtained from Hin (H_i^0)
- The position of frame j with respect to frame i (H_j^i)

such that the position of frame j with respect to the inertial frame is obtained:

$$H_j^0 = H_i^0 H_j^i \tag{16}$$

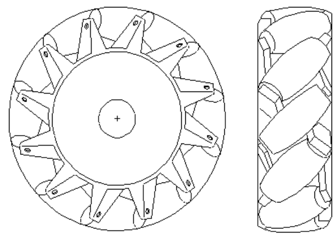

Fig. 7: An example of a Mechanum wheel (Source: www.robotika.sk)

5 Submodel of a Mecanum wheel

A Mechanum wheel is a wheel with rollers mounted on it. These rollers are mounted in an angle of 45° with respect to the wheel and can freely rotate. Fig. 7 shows an example of a Mecanum wheel.

In this paper a Mecanum wheel is considered as a construction that enables the application of forces to the robot by actuation of the wheel axis. All dynamic behavior of the Mecanum wheel is neglected for simplicity reasons.

A Mechanum wheel is, first of all, a wheel. Therefore, the model of a wheel is first discussed.

5.1 Model of a wheel

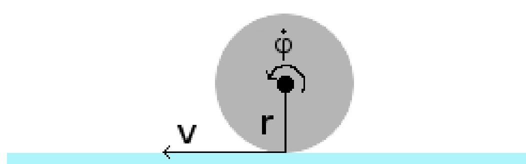

Fig. 8: A common wheel represented as transformer

An ideal wheel on a surface, as shown in Fig. 8, creates a relation between the rotational velocity ($\dot{\phi}$) of the wheel and the translational velocity (v) of the wheel with respect to the surface. It is common knowledge that the radius (r) of the wheel is a measure for this relation. More specifically:

$$v = r\dot{\phi} \tag{17}$$

On the other hand, a wheel also imposes a relation between the torque (τ) applied to the wheel and the force (F) between the wheel and the surface. The radius also is a measure for this relation:

$$F = (1/r)\tau \tag{18}$$

When comparing the energy flow, or power, on the two sides (power ports) of the wheel (translation and rotation) it is clear that it is equal:

$$P_{translation} = vF = r\dot{\phi}(1/r)\tau = \dot{\phi}\tau = P_{rotation} \tag{19}$$

In the bond-graph language this ideal behavior is described by the standard element transformer.

5.2 Extension of the model of a wheel to the model of a Mecanum wheel

At the contact with the surface, a Mecanum wheel has rollers where a common wheel has none. These rollers can rotate freely around one axis and are mounted in an angle of 45° with respect to the transformation direction of the wheel. Fig. 9 clarifies the position of a roller (grey, filled rectangle) in the wheel (black

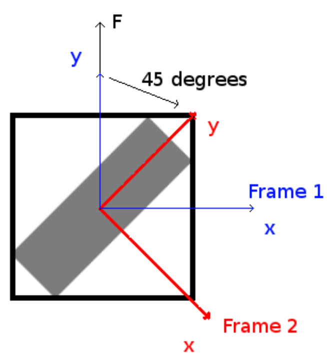

Fig. 9: Forces in frame 1 aligned with the wheel (blue) and frame 2 aligned with the roller (red)

rectangle outline). In Fig. 9 two frames are shown:

1. One frame that is aligned with the wheel, this frame will be called frame 1
2. A second frame that is aligned with the roller, this frame will be called frame 2

In Fig. 9 the axis around which the wheel rotates is the x-axis of frame 1 and the axis around which the roller can rotate is the y-axis of frame 2.

Since the roller can rotate freely around its y-axis, it acts as a bearing for translational movements in its x-direction. Just like in a common bearing the friction is minimized, for this model it is assumed that no friction is present. As a result of the lack of friction in this direction, there is no relation between:

1. the force applied by the wheel/ velocity of the wheel and
2. the force applied between the wheel and the surface/ velocity of the wheel with respect to the surface

in the x-direction of the roller. Any force applied in this direction will only contribute to the rotation of the wheel. When putting this in perspective with the model of the wheel discussed earlier, there is a transformation ratio of 0 in this x direction of frame 2. Note that this is expressed in a different frame than the transformation of the wheel, which is expressed in frame 1.

To model the effect of the rollers in the bond-graph language, a 2-dimensional transformer element is used to represent the effect of the roller in frame 2. The transformation ratio (r) of this transformer is:

$$r = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{20}$$

It has a transformation ratio of 0 in the x direction and a transformation ratio of 1 in the y directions. Note that this transformer represents a relation between two translational components.

Since the transformation of the wheel is expressed in frame 1 and the transformation of the roller in frame 2, one of the two should be expressed in the frame of the other. Such that, both transformations are expressed in the same frame. It is chosen to express the transformation of the roller in frame 1.

To express a transformer in a different frame, the transformation ratio should be premultiplied by the change of coordinate matrix A and post-multiplied by its inverse:

$$TF^j = A_i^j TF^i (A_i^j)^{-1}, \text{ where} (A_i^j)^{-1} = A_j^i. \tag{21}$$

In the bond-graph language, this change of coordinates can be represented by transformers as shown in Fig. 10.

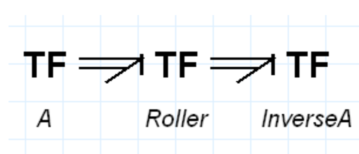

Fig. 10: Change of coordinates for a transformer in the bond-graph language

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In case of a pure rotation[7]:

$$A = \begin{pmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{pmatrix} \quad (22)$$

Where ϕ is the rotation between frame i and frame j.

When going from frame 2 to frame 1 there is a rotation of -45°, so in this case ϕ equals -45°.

Now the bond-graph model for the complete Mecanum wheel can be constructed as shown in Fig. 11.

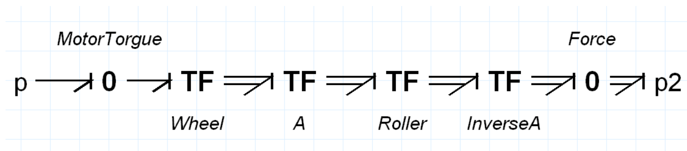

Fig. 11: The complete bond-graph model of a Mecanum wheel

Please note that in practise two types of Mecanum wheels are used:

1. One where the rollers are mounted in an angle of 45° with respect to the wheel and
2. a second type where the rollers are mounted in an angle of -45° with respect to the wheel

This difference results in slightly different models. However, both models are structured as explained in this section.

6 Construction of the model

The model of the robotic manipulator is constructed as a serial link in an object oriented way. In this model, two of the previously discussed submodel types are used:

1. submodels of links, connecting two joints
2. submodels of joints, connecting two links

Such that the model of the robotic arm is as shown in Fig. 12.

The Se elements connected to all the joints represent the torques applied by the actuators in the joints. If necessary, the model can be extended with submodels of the electric motors. Furthermore, two connections can be seen between all joints and links:

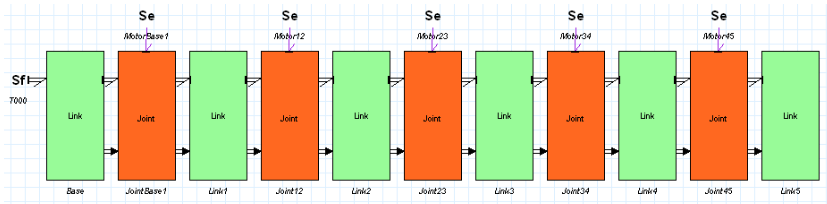

Fig. 12: Model of a robotic arm

1. a power connection establishing energetic coupling of the elements
2. a signal connection that passes on the homogeneous matrix representing the position of the connection point

So, every link and joint passes the position of the connection point with the next element on to the next element.

6.1 Connection of the Mecanum wheels to the robotic manipulator

As described in section 2, four Mecanum wheels are connected to the first link of the robotic manipulator. However, in the model they cannot be connected directly, since the model of the robotic manipulator uses screw theory, while the submodel of the Mecanum wheel does not. The model of the Mecanum wheels provides a linear force caused by the Mecanum wheel as a result of the axis actuation. This linear force is applied in a point of the link. If this would be transformed to a wrench, the wrench would be applied to the body. This is a clear advantage in addition to the earlier mentioned advantage of being able to address rotations and translations simultaneously.

Transformation from a force in a point to wrench applied to a body and from a twist of a body to a velocity of a point is power continuous: the operation does not add energy to or remove energy from the system. Transformation of a twist to a linear velocity in a point (p) is given as:

$$\dot{p} = \omega \wedge p + v \tag{23}$$

Where a twist has the following shape:

$$T = \begin{pmatrix} \omega \\ v \end{pmatrix} \tag{24}$$

This relation can be expressed in matrix form:

$$\dot{p} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 0 & z & -y & 1 & 0 & 0 \\ -z & 0 & x & 0 & 1 & 0 \\ y & -x & 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} \omega \\ v \end{pmatrix} \tag{25}$$

Based on power continuity, the relation between a force and the corresponding twist can be found:

$F^T v = F^T AT = W^T T$ so $F^T A = W^T$
when transposed on both sides :
$A^T F = W$

These two relations together are the relations of a transformer in the bond-graph language, as shown in section 5.1. So, a transformation from screws to linear velocities/ forces and vise-versa can be accomplished with a multi-dimensional transformer in the bond-graph language. Please note that the transformation transforms between a three dimensional force/ velocity and a wrench/ twist. The Mechanum wheels exert no force in the z-direction and therefore the force vector is extended to:

$$F = \begin{pmatrix} F_x \\ F_y \\ 0 \end{pmatrix} \tag{26}$$

With all the forces applied by the wheels transformed to wrenches applied to the link, they can be summed. In the bond-graph language, this is represented by a 1-junction. The summed wrench can then be applied to the base. Fig. 13 shows the complete model of the Mecanum wheels connected to the first link of the robotic manipulator. The blocks "Mecanum wheel type A" and "Mecanum wheel type B" are submodels containing the model of a Mecanum wheel as discussed earlier. The Se elements are sources of effort, they apply a fixed torque to the axis of the Mecanum wheels.

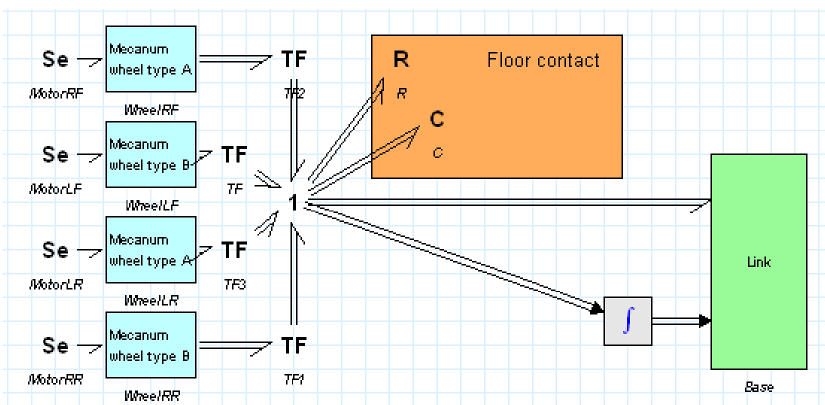

Fig. 13: Model of the Mecanum wheels connected to the first link

Since the position of the base is required in the base block, an $\int$-block has been added. This block calculates the position of the block based on the current position and the twist of the base. Furthermore, an R element (resistance) and a C element (stiffness) can be found in the model. These elements only act in the z-direction of the base and represent the contact with the floor. Since the floor

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is very stiff and has a high resistance, the values of these two components are high. Without these elements, the robot-model will accelerate in the negative z-direction due to gravity.

7 Conclusion

In this paper, a walk-trough tutorial on the modeling of a complex robotic system is presented. By writing the model in the bond-graph language, a model with reusable components has been created for an exemplary platform: youBot. In the paper it has been noted that the model, or submodels used in the model, can easily be adapted or extended with other behavior.

Validation of the model is not present in this paper since official validation still needs to be done with the actual platform. The platform was not available at the time of writing.

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